Journal of Basic and Applied Engineering Research
Print ISSN: 2350-0077; Online ISSN: 2350-0255; Volume 2, Number 15; April-June, 2015 pp. 1288-1291

http://www.krishisanskriti.org/jbaer.html

QCD Cross Section Calculation at NLO

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Abstract—*The general structure of Next Leading order (NLO) cross section calculation of quantum chromodynamics (QCD) is discussed, and general methods used to carry out practical calculation are reviewed.*

1. INTRODUCTION

Fully differential cross section is one of the important observable for the studies at high energy colliders like LHC. Reliable theoretical prediction for such differential cross section requires the inclusion of at least NLO QCD cross section. NLO calculation combines virtual one loop correction with the real emission contribution from unresolved patrons. These corrections are affected by different singularities. The ultraviolet singularities (UV) in the virtual contribution can be removed by a process called renormalization. Infrared divergences which include Soft (low momentum) and collinear (small angle) singularities are present both in real and virtual corrections these IR divergences are managed using different techniques. The real and virtual corrections have a different number of final state particles and have to integrate separately and each is Infrared divergent, only their sum is infrared finite. NLO Monte Carlo program incorporate both these pieces and allow the simultaneous comparison of many differential cross section for the particular reaction considered. However these programs require that IR singularities be eliminated before any integration can be done. There are essentially two types of methods to do this cancellation phase space slicing and subtraction method. In this paper general structure of NLO cross section is described and different methods to carry out practical NLO calculations are reviewed.

2. NLO CORRECTIONS

In NLO QCD calculation, we have to consider virtual one loop corrections and the real emission contribution from unresolved partons.Real contribution involves all Feynman diagrams with additional partons in the final state as shown in the figure1.

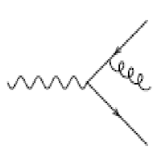

Fig. 1: Real correction

Virtual correction involves all one loop Feynman diagrams that can be obtained from born diagrams when a virtual gluon is exchanged from the quark anti quark pair such as shown in the Fig. 2.

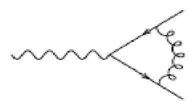

Fig. 2: Virtual correction

3. IR SAFTEY & FACTORIZATION

3.1 Infrared safety

One of the basic problems of perturbative QCD calculation is that experimentally hadrons are observed in the final state while theoretical calculations yield results for partons (quark and gluon). More over not all observable can be calculable in perturbation theory. We have to use Infrared safe observable which are constructed in such a way that all soft and collinear singularities cancel among real and virtual corrections or can be absorbed into redefined parton densities. The same observable is then evaluated both for parton final state (theoretical prediction) and hadron final states (experimental data). IR safe observable must be insensitive to emission of soft partons or to the collinear splitting of partons. This observable has to be properly defined according to KLN theorem[1][2]. It must be infrared and collinear safe or at least collinear factorizable. Such quantities are finite order by order in perturbation theory

3.2 Factorization theorem

In high-energy scattering hadrons in the initial state is in composite state. There are partons within clouds of further partons constantly being emitted and absorbed. Thus before we can use perturbatively calculated partonic scattering matrix elements, we must first address the partonic structure of the colliding hadron(s). the high momentum transfer interactions are characterized by the presence of a hard scale Q and they can be controlled through factorization theorem [3][4][5] which allows us to write the cross section as a convolution of a non-perturbative but universal (i.e., process-independent) parton distribution function (PDF)[6][7] and a perturbatively calculable short distance partonic cross section. The parton distribution functions (PDFs) parameterize the distribution of partons inside the hadron. The partonic cross section is calculable within perturbation theory; the dividing line between the two is drawn at an arbitrary ("user-defined") scale μ_F called the factorization scale. Evolution of PDFs with factorization scale can be obtained using DGLAP evolution equation [6] and can be used to run the PDFs from one scale to another. The physics is unchanged under a change of the factorization scale.

The KLN theorem [1][2] and the factorization theorems[4] constitute the theoretical basis of the description of scattering processes of hadrons in perturbative QCD. These theorems constitute the necessary consistency condition for the validity of the fundamental assumption of the QCD improved parton model. This assumption is that for the case of infrared safe quantities the perturbative QCD predictions given in terms of partons are a good approximation to the same quantities measured in terms of hadrons.

4. GENERAL STRUCTURE OF NLO

The general structure of QCD in NLO is as follows

$$\sigma = \sigma^{LO} + \sigma^{NLO} \quad (1)$$

The leading order cross section (LO) is obtained by integrating the fully exclusive cross section using born approximation $d\sigma^B$ over the phase space. Let us suppose that this LO calculation involves n partons with momenta p_k in the final state thus we can write

$$\sigma^{LO} = \int_n d\sigma^B \quad (2)$$

Where born level cross section is

$$d\sigma^B = d\emptyset^{(n)}(\{p_k\})|M_n(\{p_k\})|^2 F_J^{(n)}(\{p_k\}) \quad (3)$$

Where $d\emptyset^{(n)}$and M_n respectively denote the full phase space and the tree level QCD matrix element to produce n final state partons. These are the factors that depend on the process. The function $F_J^{(n)}$ defines the physical quantity that we want to compute this has to be infrared and collinear safe, its actual value has to be independent of the number of soft and collinear particles in the final states. Thus we should have (refer[8] for more details)

$$F_J^{(n+1)} \rightarrow F_J^{(n)} \quad (4)$$

Efficient techniques are available for calculating tree level matrix elements. Thus evaluation LO cross section does not present any difficulty.

At NLO one has to consider the exclusive cross section $d\sigma^R$ with n+1 partons in the final state and the one loop correction $d\sigma^V$ to the process with n partons in the final states.

$$\sigma^{NLO} = \int_{n+1} d\sigma^R + \int_n d\sigma^V \quad (5)$$

$d\sigma^R$And $d\sigma^V$ have the same structure as the born level cross section apart from the replacement $|M_m|^2 \rightarrow |M_{m+1}|^2$ and $|M_m|^2 \rightarrow |M_m|^2_{1-loop}$ here $|M_m|^2_{1-loop}$ denotes the QCD amplitude to produce m final state partons evaluated in the one-loop approximation. This calculation leads to ultraviolet, Soft and collinear singularities. The UV singularities can be handled by renormalization procedure[9]. Soft and collinear singularities do not cancel within the content of $d\sigma^V$and are accompanied by analogous singularities arising from the integration of the real cross section $d\sigma^R$. By adding real and virtual contribution the cross section is finite in equation (5). This is also guaranteed by equation (4). But this cancellation does not take place at integrand level.

The two integral on the RHS of cross section equation (5) are separately divergent so that before any numerical calculation the separate pieces have to be regularized, the most popular one is dimensional regularization[10]. Using dimensional regularization the divergences (arising out of integration) are replaced by double (soft and collinear)$1/\varepsilon^2$ and single (soft or collinear) pole$1/\varepsilon$. thus the real and virtual contributions should be calculated independently yielding equal and opposite poles in ε. These poles have to be combines and cancel with each other and the limit $\varepsilon \rightarrow 0$ can be safely carry out.

The real and virtual contributions have to be integrated separately over different phase space regions. Two general methods for doing this is the phase spce slicing [11]and the subtraction method[12] fisrt used in context of NLO calculation of e^+e^- annhilation then have been applied to other cross sections. Recentely it become clear that both method is generalizable in a process independent manner.the key observation is that the singulart parts of the QCD matrix elements for real emission can be singled out in general way by using factorization properties of soft and collinear radiation. Owing to this universality the two methods have led to general algorithms for NLO QCD calculations.[8][13]

5. SUBTRACTION METHOD

The general idea of the subtraction method is use the identity

$$\sigma^{NLO} = \int_{m+1}[d\sigma^R - d\sigma^A] + \int_{m+1} d\sigma^A + \int_m d\sigma^V \quad (6)$$

Which is obtained by subtracting and adding back the same quantity $d\sigma^A$ which have the same point wise singular behavior as $d\sigma^R$ itself then it act as a local counter term for $d\sigma^R$ and one can safely perform the limit $\varepsilon \rightarrow 0$ under the integral sign in the first term on the right hand side of (6). This defines a cross section contribution $\sigma^{NLO\{m+1\}}$ with $m+1$ parton kinematics that can be integrated numerically in four dimensions

$$\sigma^{NLO\{m+1\}} = \int_{m+1}[(d\sigma^R)_{\varepsilon=0} - (d\sigma^A)_{\varepsilon=0} \quad (7)$$

Analytic integrability of $d\sigma^A$ over the one parton subspace leading to soft and collinear divergences. In this case we can write the last two terms of the equation (6) as follows

$$\sigma^{NLO\{m\}} = \int_m [\, d\sigma^V + \int_1 d\sigma^A]_{\varepsilon=0} \quad (8)$$

Performing the analytic integration $\int_1 d\sigma^A$ one obtains ε-pole contributions that can be combines with those in $d\sigma^V$ thus cancelling all divergences. The remainder is finite in the limit $\varepsilon \to 0$ and thus defines the integrand of a cross section contribution $\sigma^{NLO\{m\}}$ with m parton kinematics that can be integrated numerically in four dimensions. The final structure of NLO cross section is as follows

$$\sigma^{NLO} = \sigma^{NLO\{m+1\}} + \sigma^{NLO\{m\}} \quad (9)$$

And can be easily implemented in a partonic Monte Carlo program.

The crucial part of this process is that we are free to choose the form $d\sigma^A$ so that we are able to perform the single integral analytically. It is possible to define a method of generating $d\sigma^A$ for NLO process which is process independent. Also possible for a suitable choice of $d\sigma^A$ to numerically integrate the virtual piece over the internal one-loop integral. Following these methods, it will make possible to use a completely numerical approach for any NLO observable.

There exist two general formulation of subtraction method one is the residue approach the other one is the dipole formalism. Both can handle massless partons and identified hadrons in the final and (or) initial state.

6. PHASE SPACE SLICING

This method splits up the full parton phase space into two regions a region R where all partons can be resolved and a region U where two or more partons unresolved. This splitting is usually achieved by a technical cut parameter S_{min}. Two partons with momenta P_1 and P_2 are unresolved if their invariant mass $S_{12} = 2P_1.P_2$ is smaller than S_{min} and resolved if it is larger than this value. The hard region is defined so that all the S_{12} are bigger than theoretical cut S_{min} The integration over the resolved region R can be performed numerically because all IR singularities are cut out by the phase space cut. The collinear and soft region is defined such that one or two S_{12} are smaller than S_{min}. The integration over this unresolved region U is divergent and cannot be performed numerically but because of the constraint $2P_1P_2 < S_{min}$ the cross section factorizes. In this case the calculation must be done analytically. If S_{min} is small enough, the soft and collinear approximation can be used such that the integration in n dimensions is greatly simplifies. The soft approximation is generalized to the case where the particles involved are massive. In the collinear region, the mass regularizes the singularities and the calculation can be done numerically. The poles that remain after the integration over the soft and collinear region cancel with the corresponding poles of the virtual contributions. In the context of phase space slicing method many algorithm has been developed [14][11][15]

Both methods have their merit and drawbacks. The phase space slicing method is technically simple and can easily be implemented. Once the matrix element for the real and virtual correction are known. The main problem is residual dependence on the technical cut S_{min}. The independence of numerical results from variation of this cut has to be checked; moreover the integration over the region R mentioned above requires very high statistics because the integration region is close to singular limit. Subtraction term does not require a technical cut but the construction of subtraction term is quite involved if this can be done the subtraction method is the method of choice.

7. CONCLUSION

In this paper I described the general structure of NLO QCD cross section. By using subtraction method or phase space slicing method one can extract and treat the singular parts of any NLO cross sections in a way that is independent of exact details of the observable and process. Various formulations of these two methods are available in the literature. During the last few years, effective numerical computational techniques (many automated) have been developed to calculate the fully differential cross section for NLO QCD calculations.

(some reference.[8][16][17][18][19])

8. ACKNOWLEDGEMENT

I am grateful to Dr Abbas Ali and Abdul Salih P P, department of physics, Aligarh Muslim University, Aligarh for valuable discussions and suggestions.

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